



2018 VI 12

1430

Seat No. :

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Time : 2½ Hours

MATHEMATICS AND STATISTICS (Old Pattern)

Subject Code

H	6	0	6
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Total No. of Questions : 7

(Printed Pages : 4)

Maximum Marks : 80

INSTRUCTIONS : i) This question paper contains **seven main** questions.

ii) **All seven** questions are **compulsory**.

iii) Answer **each main** question on a **fresh** page.

iv) **Use** of calculator is **not** allowed.

v) Log tables will be supplied on **request**.

vi) Graphs should be drawn on the answer paper **only**.

vii) For **each main** question, the sub-questions carry the following marks.

[A = 1 mark, B = 2 marks, C = 3 marks, D = 4 marks, E = 5 marks]

1. A) Choose and write the correct alternative from those given below :

A matrix obtained by interchanging rows and columns in a given matrix is called the _____ of the matrix.

- Adjoint
- Multiplicative Inverse
- Additive Inverse
- Transpose

B) Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

C) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find K, so that $A^2 = KA - 2I$.

D) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$, find A^{-1} , using the adjoint method.



2. A) Determine the order and degree if defined, of the following differential equation.

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0.$$

- B) Using the properties of determinants, as far as possible, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

- C) If $x = \cos\theta + \cos 2\theta$ and $y = \sin\theta + \sin 2\theta$, find $\frac{dy}{dx}$, where θ is the parameter.

- D) Solve graphically the following Linear programming problem.

$$\text{Maximise } Z = 20x + 10y$$

Subject to the constraints

$$x + 2y \leq 28$$

$$3x + y \leq 24$$

$$x \geq 0, y \geq 0.$$

3. A) Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E/F)$.

- B) Write the constraints for the following Linear Programming problem.

A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on the hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day.

- C) Find $\frac{dy}{dx}$, if $xy^2 - x^2y - 4 = 0$.

- D) If a function $f : \mathbb{R} \rightarrow \mathbb{R}$, is continuous on its domain, where

$$f(x) = 5b - 3ax \quad -4 \leq x < -2$$

$$= 4x + 1 \quad -2 \leq x \leq 2$$

$$= ax^2 + 17b \quad 2 < x \leq 4$$

Find the values of a and b.



4. A) Choose and write the correct alternative from those given below.

The difference between the resources and liabilities of a firm is called its

- Present worth
- Future value
- Profit
- Loss

B) Using the first principle, find the derivative of function $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = \log 5x$.

C) Evaluate $\int_2^7 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{9-x}} dx$.

D) A bill drawn on April 14, 2016 for 8 months after date, was discounted on July 24, 2016 at 5% per annum. If the bankers gain on the basis of simple interest is Rs. 20, what is the face value of the bill ?

5. A) Choose and write the correct alternative from those given below.

The instantaneous rate of change of revenue at any level of output is called

- Average revenue
- Cost
- Profit
- Marginal revenue

B) Find $\int x \cdot \cos x dx$.

C) Find $\int \frac{e^x}{\sqrt{(e^{2x} + 4e^x + 13)}} dx$.

D) Attempt **any one** of the following :

1) Given the cost function is $C(x) = x^3 - 57x^2 + 315x + 20$.

Find the level of output for which the cost is minimum.

2) The total revenue function is given by $R(x) = 400 + 3x + 2x^2$.

Find : 1) The average revenue

2) Marginal revenue

3) Marginal revenue when $x = 15$ units

4) Average revenue when $x = 10$ units.

6. A) Choose and write the correct alternative from those given below :

If $R = \{(a, a), (b, b), (c, c), (b, c), (a, b)\}$ is a relation on set $A = \{a, b, c\}$ then

- R is reflexive
- R is symmetric
- R is transitive
- R is an equivalence relation.

B) A and B are partners sharing profits and losses in the ratio 2 : 5. They surrender $\frac{1}{3}$ rd of each one's share in favour of C, a new partner. Calculate the new profit sharing ratio.



- C) On set R of real numbers, an operation $*$ is defined by $a*b = a + b + ab$.
Is $*$ a binary operation? Justify your answer.
If it is a binary operation, find whether $*$ is commutative and associative.
- D) X, Y and Z formed a partnership and contributed Rs. 30,000, Rs. 40,000 and Rs. 50,000 respectively towards the capital. They agree to divide the annual profit in proportion to the capital employed and to the time it is in use. After 6 months, Z withdrew Rs. 5,000 and Y added Rs. 5,000. At the end of the year, the profit was Rs. 1,22,400. How will they share this profit?
- E) Attempt **any one** of the following :
- 1) Find the future amount of an annuity of Rs. 1,00,000 payable at the end of each quarter for 6 years at 8% p.a. compounded quarterly. Use log tables.
 - 2) Find the present value of an annuity due of Rs. 700 per annum payable at the beginning of each year for 2 years allowing interest at 6% per annum compounded annually. Use log tables.
7. A) Choose and write the correct alternative from those given below.
An annuity in which payments start after a lapse of a specified number of payment periods is called _____.
- Deferred annuity
 - Ordinary annuity
 - Sinking fund
 - Annuity due
- B) Form a differential equation by eliminating the arbitrary constants A and B, in the following equation.
 $Y = Ae^{2x} + Be^{-2x}$
- C) Solve the differential equation
 $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$.
- D) Find $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$.
- E) Attempt **any one** of the following :
- 1) Find the mean number of heads in three tosses of a fair coin.
 - 2) A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced. What is the probability that it was produced by A?